Summary

The alternative derivations of Einstein and Rohrlich show that the famous formula $\boldsymbol{E = mc^2}$, usually considered a relativistic equation, is a simple consequence of the interaction between electromagnetic radiation and matter.

Thus derived, the Mass–Energy Equivalence Principle provides to the Newtonian mechanics the missing relation that allows the integration of the differential equation from the second law of dynamics in its more general form:

$$\begin{cases} dE = v^2 dm + mv dv \\ dE = c^2 dm \end{cases} \qquad \Rightarrow$$

$$c^2 dm = v^2 dm + mv dv \qquad (5.2)$$

Integrating (5.2) yields the first important relationship for the inertial mass of a body as a function of speed:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (5.4)$$

By using this relation and the laws of conservation of energy and momentum, and without the use of relativistic axiom-based hypotheses, successively succeed in deriving other important formulas attributed to the Theory of Relativity:

- The expression of kinetic energy in the more general case:

$$E_k = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2 \qquad (6.4)$$

- The equation of the total energy of the point mass:

$$mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = E_k + m_0 c^2 \qquad (6.5)$$

- The relationship between energy, mass and momentum illustrated by the so-called E-p-m triangle:

$$E = mc^2 = c\sqrt{p^2 + m_0^2 c^2} \qquad (7.1)$$

- The velocity addition formula:

$$v_{12} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} \qquad (10.6)$$

- The relativistic length contraction and time dilation depending on the speed:

$$\boldsymbol{l' = l\sqrt{1-\frac{v^2}{c^2}}} \qquad (11.6) \qquad \boldsymbol{t' = t\sqrt{1-\frac{v^2}{c^2}}} \qquad (11.7)$$

- The alternative derivation of the Lorentz transformation for space and time:

$$\boldsymbol{x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}}} \qquad (12.4) \qquad \boldsymbol{t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}} \qquad (12.7)$$

- The independence of the speed of light $\boldsymbol{v_l}$ from the relative speed $\boldsymbol{v_q}$ between emitting light source and observer:

$$\boldsymbol{v_l = \frac{c + v_q}{1 + \frac{v_q}{c}} = c} \qquad (13.1)$$

- The frequency shift for electromagnetic waves at any speed by approaching and receding of the light source:

$$\boldsymbol{f' = f\sqrt{\frac{c+v}{c-v}}} \qquad \boldsymbol{f' = f\sqrt{\frac{c-v}{c+v}}} \qquad (15.6)$$

- The relations of the longitudinal and transversal components of the relativistic acceleration as a function of the speed:

$$\boldsymbol{a_L = \frac{F_L}{m_0}\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad and \quad \boldsymbol{a_T = \frac{F_T}{m_0}\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}} \qquad (16.9)$$

All these formulas agree with those derived from the axioms of Special Theory of Relativity or rather with the use of the Lorentz transformations.