7 The Relativistic E-p-m Triangle

The relativistic E-p-m triangle illustrates in a truly clear way the relationships that exist between energy, momentum, and mass of the body at high speeds.

In the preceding sections we have seen that in many formulas the reciprocal $\sqrt{\mathbf{1}-\frac{v^2}{c^2}}$ of the so-called Lorentz factor occurs.

This term is reminiscent of the Pythagorean Theorem: According to this, if in a right-angled triangle the hypotenuse is equal to 1 and a cathetus is equal to v/c, then the second cathetus is equal $\sqrt{\mathbf{1}-\frac{v^2}{c^2}}$, as shown in Fig. 7.

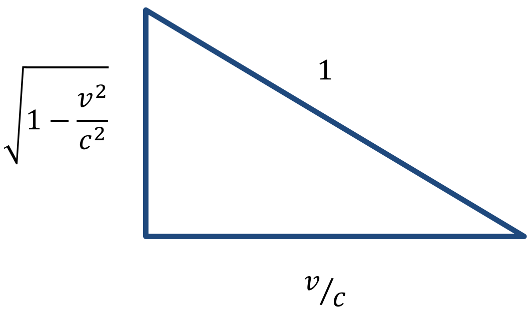

Fig. 7

Now, if all sides of the triangle are multiplied by the total energy $\boldsymbol{mc^2}$ of a body, the following results are obtained, considering the relation (5.4):

$$\text{1st Cathetus} = \boldsymbol{mc^2}\sqrt{\mathbf{1}-\frac{\boldsymbol{v^2}}{\boldsymbol{c^2}}} = \boldsymbol{m_0c^2}$$

$$\text{2nd Cathetus} = \boldsymbol{mvc}$$

$$\text{Hypotenuse} = \boldsymbol{mc^2}$$

Obtaining the so-called relativistic triangle shown in Fig. 8:

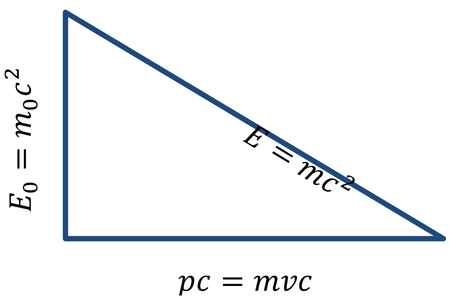

Fig: 8

If the Pythagorean Theorem is applied to the triangle in Fig. 8, then the following relations can be derived for the total energy $\boldsymbol{E}$ and the momentum $\boldsymbol{p}$:

$$\boldsymbol{E = mc^2 = c\sqrt{p^2 + m_0^2 c^2}} \qquad (7.1)$$

$$\boldsymbol{p = \sqrt{\frac{E}{c^2}^2 - m_0^2 c^2}} \qquad (7.2)$$

By the relation (7.1) it can be seen that when the momentum equals zero, the total energy is reduced to the value $\boldsymbol{m_0 c^2}$. This value corresponds to the internal energy of a point mass $\boldsymbol{m_0}$.

Equation (7.2) shows that there can be physical objects with the invariant mass $\boldsymbol{m_0}$ equal to zero.

In this case the momentum is given the value: $\boldsymbol{p = E / c}$, valid for light quanta.

Equation (7.2), however, also shows that there can be no physical objects with energy equal to zero; otherwise, the momentum would get an imaginary, i.e., not allowed value.

From the relativistic E-p-m triangle one can also gain another insight:

Consequently, if the invariant mass $\boldsymbol{m_0}$ of a particle equals zero, then the vertical triangle side is equal to zero and the horizontal triangle side becomes equal to the hypotenuse.

It follows:

$$\boldsymbol{mvc = mc^2 \quad \Rightarrow \quad v = c}$$

This means that massless physical objects inevitably move at the speed of light.

Examples of elementary particles that propagate at the speed of light are the exchange particles of the electromagnetic and gravitational interactions. That means the photons and the gravitons. In the context of Quantum Chromodynamics Theory, it is assumed that the exchange particles of the strong interaction, called gluons, also travel at the speed of light.

The results of the preceding sections can be summarized by the geometrical representation of the so-called relativistic E-p-m triangle. This triangle illustrates in a truly clear way the relationships that exist between energy, momentum, and mass.