2 Newton's Law - An Analysis

Before starting the first alternative derivation, it is worthwhile to analyze Newton's law in more detail.

As stated in the previous chapter, Newton speaks of "*Mutationem motus*" and here his intuition was right. Consequently, he has bequeathed his law to the world in the following compact form:

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \Leftrightarrow \quad \vec{F} = \frac{d(m\vec{v})}{dt} \qquad (1.3)$$

Or:

$$\vec{F} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} \qquad (2.1)$$

Obviously, the force vector consists of two terms.

In the first component we find the differential quotient of the velocity which we know as a variable vector, and nothing else is objected first.

In the second component the differential quotient of the mass occurs, and this is remarkable, since in this form the law admits the hypothetical assumption of a variable mass.

Interpreted this way, Newton's law was far ahead of its time, since in Newton's time natural phenomena involving a change in mass were unknown.

In its orbit around the sun the earth reaches a speed of about 30 km/s. This results in a ratio earth / light speed β=0.0001. The aberration constant of the planet Mercury is somewhat larger with β=0.00016. And this was probably also the highest speed Newton knew, if at all.

Today we know that at these speeds, due to the kinetic dependence of inertia, mass change is not noticeable.

Newton was thus unable to verify experimentally a possible dependence of mass on velocity, let alone determine this dependence analytically.

According to Eq. (2.1) he had two terms ($m\frac{d\vec{v}}{dt}$ and $\vec{v}\frac{dm}{dt}$) at his disposal, with which he could have explored his mechanics, but according to the knowledge of his time he used only the first one:

$$\vec{F} = m\frac{d\vec{v}}{dt} \qquad (2.2)$$

From this evolved classical mechanics which has always been a borderline case in physics and still remains so today.

So, Newton was not necessarily an optimal user of his own laws. And his idea of space and time is not correct from today's point of view.

- **Newton postulates for the time:** *"Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external…"*
- **And for the room he claims:** *"Absolute space, in its own nature, without relation to anything external, remains always similar and immovable."*

In this regard, Newton was not right.

Speed-dependent inertia of the physical body

But now we want to see whether it would have been any different if it had been possible to conclude from experiments at Newton's time that the inertia of a body is speed dependent. For this purpose, Relation (2.1) is used for a more detailed investigation in energy form.

We introduce the scalar product of the force vector $\vec{F}$ with an infinitesimal distance $d\vec{s}$ running in the direction of action of the force. An infinitesimal work is performed, transferring an infinitesimal energy:

$$\vec{F} \cdot d\vec{s} = F ds = dE$$

From (2.1) we get:

$$dE(v, m) = m\frac{ds}{dt}dv + v\frac{ds}{dt}dm$$

Or:

$$dE(v, m) = mvdv + v^2 dm \qquad (2.3)$$

In (2.3), the two terms that have now been redesigned are still clearly recognizable:

- The first one, $mvdv$, describes the speed change,
- the second one, $v^2 dm$, the mass change.

Consequently, the infinitesimal energy dE of the linear motion generally depends on both the velocity change and the mass change.

Case considerations for the speed

Now the differential equation (2.3) should be integrated. For this the relation is necessary which describes the dependence of the mass on the speed. However, this relation was unknown in Newton's time.

Today, physicists have access to particle accelerators that can be used to perform experiments at different velocities.

Case $v \ll c$: The experiments show that at low speeds the mass of the particles remains practically constant ($dm/dt \approx 0$). In these cases, the infinitesimal energy dE supplied to the body only affects the first term of equation (2.3), and therefore this relation can be simplified as follows:

$$dE(v) = mvdv + \cancel{v^2 dm} \qquad (2.4)$$

The differential equation (2.4) can be easily integrated to deduce the relation of the kinetic energy:

$$\boldsymbol{E = \frac{1}{2}mv^2}$$

And so can continue to build the classical mechanics on it.

Case $\boldsymbol{v \to c}$: At remarkably high speeds close to $\boldsymbol{c}$, however, something completely different results. The experiments show that the particles can hardly be accelerated near this speed. So, their mass seems to grow while their velocity remains nearly constant at values just below $\boldsymbol{c}$. It follows $\boldsymbol{dv/dt \approx 0}$ even with not excessively large values of the mass. Thus, the term $\boldsymbol{mvdv}$ becomes negligibly small compared to $\boldsymbol{v^2dm}$.

At very high speeds near $\boldsymbol{c}$ (2.3) becomes (2.5):

$$\boldsymbol{dE(m) = \cancel{mvdv} + v^2dm} \qquad (2.5)$$

This shows that in this case the increase in energy practically only affects the second term of relation (2.3). This causes the mass of the system to increase at an almost constant speed.

From (2.5) for $\boldsymbol{v \to c}$ the following equation can then be derived:

$$\boldsymbol{dE = c^2dm} \qquad (2.6)$$

... which, in infinitesimal form, corresponds to the principle of equivalence between energy and mass $\boldsymbol{\Delta E = \Delta mc^2}$. However, this only applies to $\boldsymbol{v \approx c}$ in this example and is not a general proof of the equivalence principle.

And what can one conclude from this?

The experimental results and the first term of Newton's law give a correct description of the natural phenomena at low speeds, as it is still the case in classical mechanics.

However, the experiments also show that the body's inertia does not remain constant at high speeds, with the following consequences:

- The mass must not be viewed as a constant.
- There is an upper limit on the speed.
- The Galileo transformation is useless for high speeds.
- The second law of dynamics must be used with the two terms that result from its differentiation, so that it remains generally valid for any speed.

With this last knowledge we can ask ourselves the question:

If only classical mechanics can be derived from the first term of the relation (2.1), what can be achieved by using both terms?

And Newton would certainly have asked himself this question if the results of experiments at high speeds had been available to him.

A concrete answer to this question can be considered as the main task of the present work.

Look back into the past:

At that time Newton did not have any experimental results at high speeds available, so he assumed that the mass remains constant at any speed. Consequently, he used only one term.

The physicists, his successors, inherited both terms from him but used only one, even after the results of experiments at high speeds became available to them. And so, the conviction continued to prevail that Newton's law applies only at low speeds and immutable mass.

To this day, this attitude to Newtonian mechanics has not changed.

Wikipedia is a reliable indicator of scientific opinion.

In the English version of the article on Newton's laws of motion, it is asserted that the relation of the second law of motion $\boldsymbol{\vec{F} = d(m\vec{v})/dt}$ holds only for constant mass:

"*[...] Since Newton's second law is valid only for constant-mass systems, m can be taken outside the differentiation operator by the constant factor rule in differentiation.[...]* " (Status: Nov. 2018).

Other physicists, however, believe that although Newton's second law was conceived for constant mass only, but it can be corrected by a "*relativistic intervention*".

For example, Richard Feynman writes on Newton's second law of motion in his work "Lectures on Physics" (Chapter 15):

> „*For over 200 years the equations of motion enunciated by Newton were believed to describe nature correctly, and the first time that an error in these laws was discovered, the way to correct it was also discovered. Both the error and its correction were discovered by Einstein in 1905.*
> *Newton's Second Law, which we have expressed by the equation*
> $$F = d(mv)/dt,$$
> *was stated with the tacit assumption that m is a constant, but we now know that this is not true, and that the mass of a body increases with velocity. In Einstein's corrected formula m has the value*
> $$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$
> *where the rest mass m_0 represents the mass of a body that is not moving and c is the speed of light [...]".*

In fact, when in equation $\boldsymbol{F = d(mv)/dt}$ the mass $\boldsymbol{m}$ is replaced by the formula $\boldsymbol{m_0/\sqrt{1 - v^2/c^2}}$ of the relativistic velocity-dependent mass, after differentiating we get the expression of the relativistic acceleration (see A II in the Appendix).

This confutes the opinion of those who claim that Newton's law is applicable only to immutable mass.

But that is not all. In the course of this work, it will be shown that the second law of dynamics remains generally correct, even without the "relativistic intervention" mentioned above. Because by Newton's law without relativistic assumptions exactly the speed-dependent formula $\boldsymbol{m = m_0/\sqrt{1 - v^2/c^2}}$ can be derived for mass without relativistic assumptions (see chapter 5).

But let us now return to differential equation (2.3).

We have analyzed the two limiting cases where only one of the two terms of (2.3) is appropriate to describe the physical events sufficiently accurately. These are the cases: $\boldsymbol{v << c}$ and $\boldsymbol{v \to c}$.

But what if the speed is between these two limits, e.g., about in the middle?

Then both terms of relation (2.3) must be used, and thus the mechanics are described using both. Thus, both velocity and mass changes are considered in one formula, and we shall see that Newton's law remains valid.

This is the essential result that we would like to show the reader with this study.

Definitions

To avoid misunderstandings in this work, we will use the following names for the physics sections and their results:

- *"Classical mechanics"* refers to the part of mechanics that only works with the first term of relation (2.3).
- "*Newtonian mechanics*" is the part of mechanics that uses both terms of relation (2.3). We will see that, starting from it, the formulas of Special Theory of Relativity can be derived alternatively.
- "*Classical physics"* are those areas of physics that do without the concepts of quantum mechanics and without the direct or indirect use of Lorentz transformations.
- "*Relativistic derivations*" refers to the proofs of formulas which directly or indirectly use the Lorentz transformations.
- *"Non-relativistic* or *alternative"* derivations are the proofs that **do not require** direct or indirect use of the Lorentz transformations.
- The results of "*classical physics"* are all formulas and methods that can be theoretically proven for significantly lower velocities than those of light and / or can be experimentally verified for $v \ll c$. **This includes the formulas of energy, momentum, and Doppler Effect for electromagnetic waves at low speeds**.

These results (and only them) are used in this work to derive the formulas of the Special Theory of Relativity alternatively.