1. About the Limitations of Classical Mechanics

The limitations of classical mechanics for the explanation of the phenomena of nature are referred to at the beginning of numerous works on the Theory of Relativity. Analogously, these works often proceed as follows.

> Most treatises are based on experiments that prove the immutability of the speed of light in all inertial systems, regardless of their state of rest or movement.
>
> As observers in different frames of reference, despite their relative movement to each other, measure the same speed of light, the consequence is that they cannot agree on the simultaneity of events or on the dimensions of bodies.
>
> Based on these findings, one is forced to say goodbye to the idea of an absolute space and an absolute time.
>
> But since space and time are relative quantities, there arises the necessity of redefining the relativity principle of mechanics.
>
> As a result of this development, coordinate transformations between reference systems and the measurements of space[1] and time observed in them are determined as a function of their velocities.
>
> It is then confirmed that all these hypotheses are consistent with the experimental observations at high speeds.
>
> Finally, attention is drawn to the shortcomings of classical mechanics not only in the cosmic domain but also in the experimental field of subatomic particles.

This is essentially what is said in the publications on the theory of relativity about classical mechanics. But to prove the incompleteness of classical mechanics, one can also follow a different, simpler path than the one just described.

Newton's second law of motion, which gives rise to classical mechanics, is usually expressed by the following relation between the force vector and the acceleration vector:

$$\vec{\boldsymbol{F}} = \boldsymbol{m}\vec{\boldsymbol{a}} \quad \Leftrightarrow \quad \vec{\boldsymbol{F}} = \boldsymbol{m}\frac{\boldsymbol{d}\vec{\boldsymbol{v}}}{\boldsymbol{dt}} \qquad (1.1)$$

The proportionality constant $\boldsymbol{m}$, called mass, represents a measure of the inertia of the rigid body on which the force acts.

Starting from the relation (1.1), the energy transferred to the mass $\boldsymbol{m}$ by the force $\boldsymbol{F}$ along the infinitesimal path element $\boldsymbol{ds}$ can be expressed by the following differential equation:

$$\boldsymbol{Fds} = \boldsymbol{m}\frac{\boldsymbol{dv}}{\boldsymbol{dt}}\boldsymbol{ds} = \boldsymbol{mvdv} \qquad (1.2)$$

[1] These are the so-called Lorentz transformations. They fulfill the central requirement of the Theory of Relativity: the same physical laws must be valid independent of the relative speed between the reference systems.

Later we will see that the integration of the relation (1.2) results in the kinetic energy of the mass point.

It should be noted that the equations (1.1) and (1.2) presuppose the direct proportionality between force and acceleration of a point mass.

Consequence of this approach is, at changes of speeds, inertia of mass point, on which force affects, is unchanged.

If the fictitious mass point is replaced by real atomic particles, the assumption of the immutability of inertia in the classical view means that, for example, an electron can be accelerated by a strong electric field up to an arbitrarily high velocity.

However, this hypothesis turns out to be wrong for speeds approaching to the speed of light, as experimental observations in particle accelerators show.

The experiments show that an increase in particle inertia can be detected with increasing speed. This leads to a progressive decrease of the particle acceleration, although an unchanged constant acting force should cause a constant acceleration and thus a further increase of the velocity.

For speeds approaching those of light, acceleration even approaches zero.

The velocity therefore remains almost constant, which means that for high speeds a direct proportionality between force and acceleration no longer exists.

Does this mean that the basic law of mechanics is wrong?

Not at all!

The second law of motion, as Newton formulated it, is correct.

An old physics book of mechanics aptly states:

"The original formulation of Newton's second law of motion is as follows: the force acting on a body is equal to the time derivative of the momentum vector."[2]

In his famous work "Philosophiae Naturalis Principia Mathematica" Newton writes:

"Mutation motus proportional esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur."

So, *"Mutationem motus"* ... and not *"Mutationem velocitatis"*.

This can be interpreted as follows:

[2] Daniele Sette - Lezioni di fisica - Volume I, page 100, Nov. 1963. It would be possible to discuss what Newton meant exactly. Important for this work is the correct interpretation of his law.

$$\vec{\boldsymbol{F}} = \frac{\boldsymbol{d}\vec{\boldsymbol{p}}}{\boldsymbol{dt}} \quad \Leftrightarrow \quad \vec{\boldsymbol{F}} = \frac{\boldsymbol{d}(\boldsymbol{m}\vec{\boldsymbol{v}})}{\boldsymbol{dt}} \qquad (1.3)$$

Equation (1.3) is the general interpretation of Newton's second law of motion and should be used for a correct application of mechanics, rather than the relation (1.1).

In equation (1.3) $\vec{\boldsymbol{p}}$ represents the momentum vector of the rigid body. It is identical to the product $\boldsymbol{m}\vec{\boldsymbol{v}}$ of mass and velocity.

It is obvious that (1.3) can be reduced to (1.1) if the mass remains nearly unchanged, which is certainly valid for $\boldsymbol{v} \ll \boldsymbol{c}$.

For velocities approaching the speed of light, however, mass must be considered as a function of velocity, and therefore differentiation is required.

If the elementary work is expressed by the force $\boldsymbol{F}$ along the infinitesimal path element $\boldsymbol{ds}$, starting from (1.3) and analogously to (1.2) the following differential equation[3] results:

$$\boldsymbol{Fds} = \frac{\boldsymbol{ds}}{\boldsymbol{dt}}\boldsymbol{dp} \quad \Leftrightarrow \quad \boldsymbol{Fds} = \boldsymbol{vd}(\boldsymbol{mv}) \qquad (1.4)$$

Respectively:

$$\boldsymbol{dE} = \boldsymbol{Fds} = \boldsymbol{mvdv} + \boldsymbol{v}^{\boldsymbol{2}}\boldsymbol{dm} \qquad (1.5)$$

The relation (1.5) is of fundamental importance for the objective of the present work.

It should be noted that Eq. (1.5) is simplified to (1.2) if the mass, and thus the inertia of the body, remains unchanged ($\boldsymbol{dm} = \boldsymbol{0}$).

In contrast to relation (1.2), the relationship (1.5) takes into account that a transfer of energy to a rigid body is accompanied by an increase in its inertia, as evidenced by experimental observations at high speeds. Thus, equation (1.5) provides the fundamental energy relation to the further development of Newtonian mechanics for arbitrary velocities.

In the course of this work, we will therefore refer to relation (1.5) for the following alternative derivations:

- the relativistic formula of the velocity dependence of mass (chapter 5)
- the kinetic and the total energy of the rigid body (chapter 6)
- the relationship of relativistic acceleration depending on speed (chapter 16).

From relation (1.5) all other relativistic formulas considered in this paper can also be derived indirectly.

Since the relation (1.5) contains three unequal differentials (ds, dm, dv), it can only be integrated if a second relation between energy and velocity or energy and mass is known.

[3] From now on, the formulas dispensed with the use of vectors, and instead their amounts are used in the assumption that the path ds is always parallel to the force F.

By such a relation it would then be possible to eliminate one of the three differentials from (1.5) and thus be able to carry out the integration.

The next sections will provide the missing relation and thus allow the integration of the differential equation (1.5).

> The most commonly used relation of the second law of dynamics is based on the direct proportionality between force and acceleration. In this form, Newton's law is not suitable for describing physical relationships at high velocities where an increase in body inertia is observed. Correctly interpreted as a temporal derivation of the momentum, the second law of dynamics, however, gains the character of a universally valid law and thus provides the suitable expression that takes mass change into account. However, the resulting differential equation cannot be solved without the second relation between energy and mass.