15 Dependence of Frequency on Velocity

In this chapter we will investigate the change in the frequency of the electro-magnetic radiation as a function of the speed.

For this, we again refer to phases II and III of the physical process considered in the fourth chapter, where the annihilation of one particle and the subsequent emission of two photons are described.

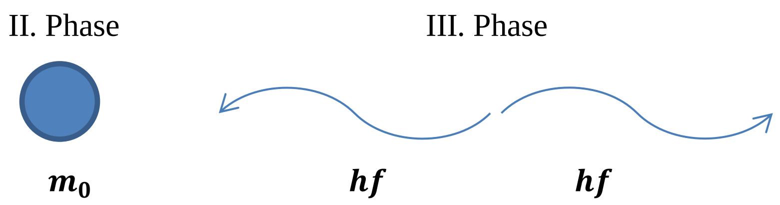

Fig. 22

We assume an observer moving in the same direction of a photon at a low speed relative to the particle.

As already stated in the third chapter, he will then measure the following frequency shift due to the optical Doppler Effect [1]:

$$f' = f\left(1 \pm \frac{v}{c}\right) \qquad (15.1)$$

However, if the velocity of the observer is close to that of the light, then it turns out that the expression (15.1) is no longer correct.

Therefore, to calculate the frequency change as a function of the velocity in the general case, we will apply the law of conservation of energy and momentum to phases II and III of the described natural phenomenon.

Since the entire mass of the particle transforms into the energy of the photons, the following applies:

$$m_0c^2 = 2hf \qquad (15.2)$$

If f_1 and f_2 represent the frequencies measured by the observer in or against the direction of motion, the following equation can be established, because of the energy conservation, before and after the annihilation of the particle:

$$mc^2 = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = hf_1 + hf_2 \qquad (15.3)$$

The application of the law of conservation of momentum also results in:

[1] The following relation (12.1) is experimentally verifiable with the measuring instruments available to physicists nowadays.

$$\boldsymbol{mv} = \frac{\boldsymbol{m_0 v}}{\sqrt{\boldsymbol{1 - \frac{v^2}{c^2}}}} = \frac{\boldsymbol{hf_1}}{\boldsymbol{c}} - \frac{\boldsymbol{hf_2}}{\boldsymbol{c}} \qquad (15.4)$$

If the expression $\boldsymbol{2hf/c^2}$ from equation (15.2) is substituted into formulas (15.3) and (15.4) instead of $\boldsymbol{m_0}$, the following system of equations results from algebraic simplification:

$$\begin{cases} \boldsymbol{f_1 + f_2} = \dfrac{\boldsymbol{2f}}{\sqrt{\boldsymbol{1 - \frac{v^2}{c^2}}}} \\ \boldsymbol{f_1 - f_2} = \dfrac{\boldsymbol{2f \frac{v}{c}}}{\sqrt{\boldsymbol{1 - \frac{v^2}{c^2}}}} \end{cases}$$

Solving for $\boldsymbol{f_1}$ and $\boldsymbol{f_2}$, we get:

$$\boldsymbol{f_1} = \boldsymbol{f} \frac{\boldsymbol{(1 + \frac{v}{c})}}{\sqrt{\boldsymbol{1 - \frac{v^2}{c^2}}}} \quad ; \quad \boldsymbol{f_2} = \boldsymbol{f} \frac{\boldsymbol{(1 - \frac{v}{c})}}{\sqrt{\boldsymbol{1 - \frac{v^2}{c^2}}}}$$

Or in a compact form:

$$\boldsymbol{f'} = \boldsymbol{f}\left(\boldsymbol{1} \pm \frac{\boldsymbol{v}}{\boldsymbol{c}}\right)\boldsymbol{\gamma} \qquad (15.5)$$

Where $\boldsymbol{\gamma}$ represents the so-called Lorentz factor. It should be noted that the relations (15.1) and (15.5) are identical except for this factor, and that the classical formula (15.1) represents a limiting case of the relativistic relation (15.5) for $\boldsymbol{v \ll c}$ (and consequently for $\boldsymbol{\gamma = 1}$).

By simple algebraic transformations, the following expressions finally result for the frequency shift in, or contrary to the direction of movement, dependent on the velocity:

$$\boldsymbol{f_1} = \boldsymbol{f}\sqrt{\frac{\boldsymbol{c + v}}{\boldsymbol{c - v}}} \quad ; \quad \boldsymbol{f_2} = \boldsymbol{f}\sqrt{\frac{\boldsymbol{c - v}}{\boldsymbol{c + v}}} \qquad (15.6)$$

The equations (15.6) agree with the relativistic formulas of the frequency shift.

In Figures 23 and 24, the violet curve shows the classical course and the green curve the relativistic course of the frequency shift.

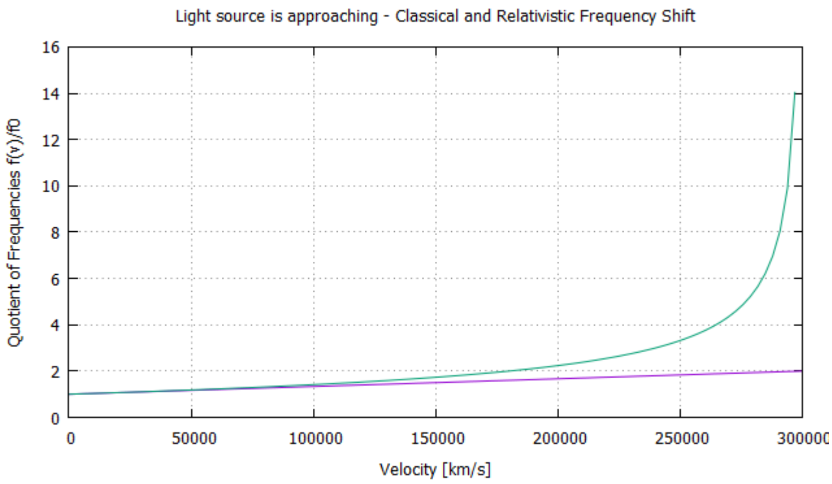

Fig. 23

According to the classical formula of the frequency shift (15.1) for a light source approaching the observer (Fig. 23), the frequency can maximally double up to the speed of light, while according to the relativistic formula (15.6), for high speeds of the light source, the frequency tends towards infinity.

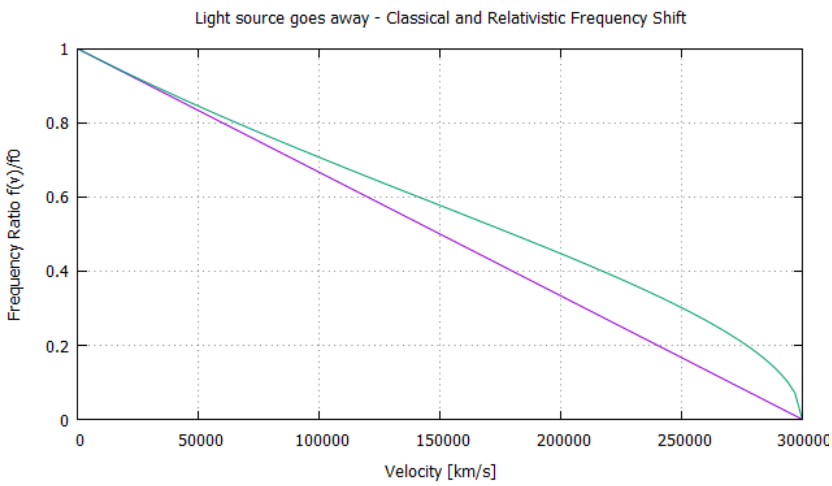

Fig. 24

If, instead, the light source moves away from the observer, the difference between the curves of formulas (15.1) and (15.6) is no longer relevant (Fig. 24).

In this case, it is also interesting to note that not only the relativistic relation (15.6), but also the classical (15.1) indicates that the speed of light cannot be exceeded, because the equation $\boldsymbol{f' = f(1 - v/c)}$ with $\boldsymbol{v > c}$ yields a negative and therefore impermissible frequency value.

In this respect, however, the classical equation (15.1) does not possess the sharpness of the relativistic relation (15.6), which is defined mathematically neither for $\boldsymbol{v > c}$ nor for $\boldsymbol{v = c}$.

> The application of the momentum and energy conservation law to the experimental observation of electron-positron annihilation makes it possible to determine the frequency shift of the electromagnetic radiation as a function of the velocity of the emitting source.