4 The Equivalence of Mass and Energy

The derivations of the previous chapter describe an important aspect of the transformation of mass into energy. Nevertheless, they do not constitute complete proof of the equivalence principle of energy and mass.

In fact, the way in which the relation (3.7) was obtained proves that in electromagnetic emission a mass fraction of a body can be converted into energy. The equation (3.7) does not prove, however, that the entire mass of a body can be converted into energy, nor that conversion into forms of energy other than electromagnetic energy is possible.

The relation (3.7) does not therefore confirm the equivalence of mass and energy in general.

The purpose of this section is to fill this gap by considering the experimental "electron-positron annihilation" observation.

This natural phenomenon can be reproduced in special particle accelerators called storage rings.

It is the reaction that can occur through the collision of the electron with the positron, the electron's antiparticle of equal mass and opposite charge:

$$\boldsymbol{e^- + e^+ \rightarrow 2\gamma}$$

As a result of the collision, an unstable particle may form for a very short time.

The decay of this particle can then generate two photons that are emitted in opposite directions.

Fig. 4 illustrates the three phases of the physical process just described:

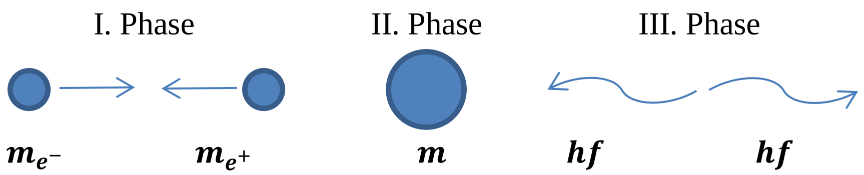

Fig. 4 (see the animation)

This phenomenon is like the thought experiment described in the third chapter. The main difference, however, is that in this case, not just one part, but the whole mass of a particle turns into energy.

We now assume an observer moving at velocity $\boldsymbol{v \ll c}$ relative to the particle formed by the collision of electron and positron.

We also assume that the direction of movement of the observer is the same as that of one of the two photons.

By applying the law of conservation of momentum before and after annihilation (phases II and III), from the observer's point of view, considering the optical Doppler Effect, the following equation can be established:

$$\boldsymbol{mv = \frac{hf}{c}\left(1 + \frac{v}{c}\right) - \frac{hf}{c}\left(1 - \frac{v}{c}\right)}$$

This can be simplified as follows:

$$\boldsymbol{mv = 2\,\frac{hfv}{c^2}}$$

Since $\boldsymbol{2hf}$ is equal to the radiated energy $\boldsymbol{E}$, we get:

$$\boldsymbol{m = \frac{E}{c^2} \quad \Leftrightarrow \quad E = mc^2} \qquad (4.1)$$

It should be noted that in relation (4.1), unlike Δm in (3.7), m is now not just a mass fraction, but represents the total mass of a particle that has completely converted into energy.

From this it can be concluded that every physical object has an internal energy that can be expressed by the relation (4.1).

This insight gives us the opportunity to establish the following energy balance of the three phases of the experiment just described:

If $\boldsymbol{m_e}$ represents the mass of the electron, then (4.1) holds for its internal energy: $\boldsymbol{E_e = m_e c^2}$.

- For the first phase before the collision, the system consisting of the electron-positron pair has a total energy equal to $\boldsymbol{2m_e c^2 + 2E_k}$, where $\boldsymbol{E_k}$ represents the kinetic energy of a single electron.
- Because of the law of conservation of energy, the total energy of the first phase will be transformed into the internal energy $\boldsymbol{mc^2}$ of the unstable particle formed by the collision (Phase II).
- After the collision, this energy finally passes into the electromagnetic energy of the two emitted photons (Phase III).

For the total energy of the system in the three phases described, the following relation can be established:

$$\boldsymbol{E = 2m_e c^2 + 2E_k = mc^2 = 2hf} \qquad (4.2)$$

The relation (4.2) confirms the conversion of mass into kinetic as well as electromagnetic energy in a particularly significant case.

It should be noted that both $\boldsymbol{m_e c^2}$ and $\boldsymbol{mc^2}$ represent only the internal energies of the particles at rest. A relation of the total energy of a particle as a function of the speed has not yet been derived.

In addition, it should be pointed out that both the phenomenon of annihilation just described, and the opposite process of so-called electron-positron pair formation occur.

As a result, an electron and a positron are formed by the decay of a photon with a minimum energy of $1{,}02\ MeV$.

For higher energies, an increase in the kinetic energy of the particles produced is observed. This confirms the general possibility of converting energy to mass and vice versa of mass to energy.

> In this section, the experimental observation "electron-positron annihilation" was considered. Through this physical process, the dissolution of the affected particles and the subsequent emission of two photons can be observed. The analysis of the phenomenon enables us to prove the principle of equivalence mass-energy in the general case. Among other things, the transition of kinetic energy into mass, as well as the complete conversion of the mass of a particle into radiant energy is investigated. These results make it possible to assign an internal energy to a particle at rest according to the equation $E = mc^2$. However, they do not yet provide the formula for the total energy of a point mass as a function of velocity.