Historia Operis

During my years as a student, I started to calculate an alternative derivation of the equation $E = mc^2$.

I knew that this equation is related to the Theory of Relativity. But I also knew that it holds for all phenomena in the context of classical physics unchanged[1]. Why should she then be provable only "relativistically" and not "classically"?

I first tried to prove the formula with classical physics using the law of conservation of energy and the physical properties of electromagnetic radiation. But I failed again and again in the calculation of the derivation.

Since the relation $E = mc^2$ concerns energy, I tried to solve the problem by relying on evidence based on energy balance. And so my attempts were always unsuccessful until the day I read the following sentences in Wikipedia about the famous formula:

“An alternative version of Einstein's thought experiment was proposed by Fritz Rohrlich (1990), who based his reasoning on the Doppler Effect. Like Einstein, he considered a body at rest with mass M. If the body is examined in a frame moving with nonrelativistic velocity v, it is no longer at rest and in the moving frame it has momentum P = Mv.”

After reading these lines I suddenly realized: In the derivation I should not use the energy balance and not the law of conservation of energy, but an equation based on the conservation of momentum!

Based on this information, I began to search for the solution myself.

Finally, I succeeded to the goal.

Chapters 3 and 4 list three alternative derivations of the equivalence principle E-M. Two of these derivations are based on the optical Doppler effect or the frequency shifts of electromagnetic radiation.

This sentence, unlike the acoustic Doppler Effect, has two important properties that, in a sense, already link it to the Theory of Relativity:

$$f' = f(1 \pm v/c) \qquad (15.1)$$

First, v in the frequency shift formula (15.1) represents the relative velocity between the light source and the observer. That is, it does not matter if the *light source, the observer, or both are moving. Every observer can consider himself as being at rest.*

Thus, the classical relation (15.1) for the propagation of electromagnetic waves, as well as the relativistic formula, does not include any physical dependence on a carrier medium, let alone on an absolute space.

1 In all exothermic reactions, a mass decrease (or mass defect) of the reaction products compared to the starting materials can be determined. This mass defect can be explained by a conversion of mass into energy according to the Mass-Energy Equivalence Principle. This applies to both nuclear and chemical reactions. Many of these reactions can only be described by classical physics.

Furthermore, equation (15.1) shows that for $v > c$ the frequency would have a negative and therefore impermissible value.

After this first step, a few years passed until the day I realized that the relation of the principle of equivalence energy-mass can be used to solve the differential equation of mechanical work (see relation 1.5).

This led to the alternative derivation of the dependence of mass on the velocity that is described in the fifth chapter of this thesis.

Remarkable in this derivation is the appearance of the Lorentz factor $1/\sqrt{1-v^2/c^2}$ in the mass formula (see relation 5.4). This factor is always considered to be an exclusive result of the Lorentz transformations, but they are not used in this derivation.

- Just coincidence? - and - Is this just a unique sense of achievement? - I asked myself then.

But one thing was clear to me: With this derivation it had become apparent that there was more potential in the "Lex Secunda" of Newton than I had previously suspected.

In its complete form, that is, with both terms (see relation 2.1), Newton's law had led to this particularly important relativistic relation.

Was that any indication that you could do any more with it?

I wondered if the second term $\boldsymbol{\vec{v}\frac{dm}{dt}}$ of relation (2.1) not at all enables the *missing link* between Newtonian and relativistic mechanics.

To test this, I had to try to work with Newton's law in connection with what had been achieved so far.

I now had two important relativistic formulas at my disposal, but they had been derived from a purely classical point of view.

The **first** was the relation of the equivalence of the energy and mass:

$$\Delta E = \Delta m c^2 \qquad (2.6)$$

The **second** was the formula of mass dependence on speed:

$$m = m_0/\sqrt{1-v^2/c^2} \qquad (5.4)$$

I knew that the integration of the differential equation $dE_k = m_0 v dv$ from the second law of dynamics (see relation 1.2) leads to the calculation of the kinetic energy in classical mechanics:

$$E_k = m_0 \int_0^v v\, dv = \frac{1}{2} m_0 v^2 \quad (6.1)$$

It was clear to me that the kinetic energy (6.1) can only apply to low velocities and immutable mass, because it is derived solely from the first term $\boldsymbol{m_0 v d v}$ of the differential equation of the work. However, in its full form, it must be defined as follows:

$$dE_k = mvdv + v^2 dm \qquad (1.5).$$

If I wanted to get a valid relation of kinetic energy for arbitrary velocities, I had to integrate (1.5). However, this task is not readily possible because $\boldsymbol{m}$ in (1.5) is speed-dependent unlike m_0 in (1.2).

But now this dependence could be removed by using the two additional formulas (2.6) and (5.4).

Their substitution in (1.5) then led to the differential equation (6.3), whose integration yielded the formula of relativistic kinetic energy.

This was yet another particularly important formula of the Theory of Relativity derived from the "Lex Secunda".

There was no doubt about it:

The application of the second law of dynamics with its full content led directly to the Theory of Relativity and the second term $\vec{\boldsymbol{v}}\frac{\boldsymbol{dm}}{\boldsymbol{dt}}$ of (1.5) seemed to be the *link* to it.

A similar procedure as for kinetic energy was used for the derivation of the acceleration for arbitrary velocities (see chapter 16)

The result was the confirmation of another relativistic formula without the use of the Lorentz transformation, which is currently the basis of relativity theory.

As an important link in the chain of proof up to the constancy of the speed of light, all that was missing was the derivation of the relativistic addition formula for velocities.

At first, I introduced a thought experiment, in which two electrons traveling at the same speed collide.

By using the mass formula (5.4) and the energy conservation law, I then provided a first proof of the addition formula for <u>equal velocities</u> (see chapter 8).

After this, with a similar method as in chapter 8, I started looking for an alternative derivation of the relativistic addition formula for velocities, but this time for <u>arbitrary velocities</u>.

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The calculation led to a rather complicated algebraic derivation with many terms, which gave hardly any hope to reach the correct result.

In the middle of the calculation, however, the equations became increasingly simplified as if by magic (see equations after relation 10.5) and then led to the relativistic formula of the velocity addition.

I finally arrived!

This last proof showed that the constancy of the speed of light, as a principle of special relativity, is not a postulate but a principle provable with the help of Newtonian mechanics and the validity of classical physics.

Thus, a thoroughly traceable transition from Newtonian to relativistic mechanics could be demonstrated.

And yet I was not quite satisfied.

With the sequence of alternative derivations, I had indeed achieved the principle of the constancy of the speed of light. According to this principle the Lorentz transformations can be derived by two different methods, as Max Born shows.

The first method uses a rather complicated and therefore difficult to follow geometric derivation.

The second method uses a much simpler algebraic method, which is based on the rather arbitrary assumption that the desired transformation is linear and therefore differs from the Galilean transformation only because of a factor to be calculated (it is the Lorentz factor).

Both derivations do not agree with the method used in my work, which instead uses conservation principles to prove all other relativistic formulas.

The problem was related to the following question:

How can length contraction or time dilation be derived for a reference system in relative motion?

In Chapter 9 I had shown that there must be a dependence of time on speed, since two observers do not agree on measured time intervals.

However, the thought experiment used did not make it possible to determine this dependence correctly. Starting from the arbitrary assumption that the observers measure the same lengths, the formulas obtained for time did not agree with the relativistic ones.

On the other hand, I was faced with a seemingly insoluble task, because to calculate the dilation of time correctly, I had to know the contraction of space, which, however, could only be calculated if the temporal dilation was known beforehand.

To solve the problem, I had to use a thought experiment in which the observers could agree either on the time measurement or on the length.

The way there leads through the observation of one-dimensional processes by two observers moving on an orthogonal axis and thus taking the same measurements for distances, velocities and times, as it also results from the Galilean transformation.

The thought experiment described in Chapter 11 allows observers moving orthogonally to the collision course of the particles to define a "common" time interval that elapses until both particles collide.

The consideration of a possible time dilatation is thus omitted and the length contraction in the direction of motion of the observers results directly from the energy and impulse conservation laws.

Using the formula of length contraction as a function of velocity, it is then very easy to derive the Lorentz transformation, as shown in Chapter 12.

This last step thus completes the alternative derivation of the special theory of relativity.