9 Dependence of Time on Velocity

The finding from the previous section that velocities cannot be added algebraically leads to two important consequences.

The first consequence concerns the failure of the Galilean transformation.

Applied to the addition of high speeds, the Galilean transformation leads to false results.

It recognizes the need to define another transformation that remains valid even at high speeds.

The second consequence concerns the time.

What it is about, we will show by the following physical examination.

Reference is now made to the thought experiment described in chapter 8 to explore the times measured by two observers in relative motion against each other.

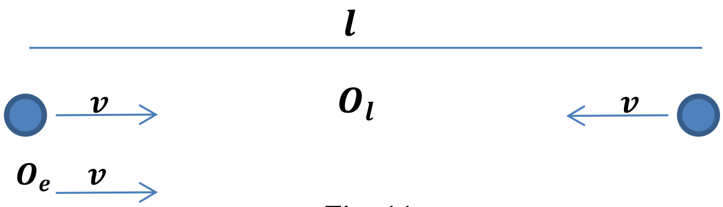

Fig. 11

We assume that in a synchrotron two particles are accelerated towards each other until their velocity $\boldsymbol{v}$ approaches the speed of light.

We imagine that at some point the particles are at the entrance of the particle accelerator detector, where they move straight up to collision at a constant velocity.

And now we adopt two assumptions, which are self-evident in the context of classical mechanics, but which are not consistent at high speeds, as in the present case, and let's see what happens.

The first wrong assumption is about space. We assume the *invariance of the lengths* for observers in relative motion: We assume that the observers $\boldsymbol{O_l}$ and $\boldsymbol{O_e}$ agree on the length of the detector, i.e., about the distance the particles will travel before the collision.

The second wrong assumption is about time. We assume the *invariance of simultaneity* for observers in relative motion to one another: We assume that from the point of view of both observers, the particles are at the same point in time in the entrance of the particle accelerator detector.

The observers now calculate independently of each other the time that elapses between the entry into the detector and the collision of the particles.

For observer $\boldsymbol{O_l}$, because he is at rest from the experimental laboratory, the particles meet exactly in the middle of the detector and therefore he finds that a single particle travels the path length $\boldsymbol{l/2}$ before the collision.

The time measured by him is therefore:

$$\boldsymbol{t_l = \frac{l}{2v}}$$

On the other hand, observer $\boldsymbol{O_e}$, being at rest at one of the particles (see Fig. 11), calculates the time by dividing the length $\boldsymbol{l}$ of the detector by the velocity at which the second particle approaches him.

We saw in chapter 8 that this velocity was calculated by the relation (8.6).

The time measured by $\boldsymbol{O_e}$ is therefore:

$$\boldsymbol{t_e = \frac{l}{2v}\left(1 + \frac{v^2}{c^2}\right)}$$

And thus, divergent from the time measured by the observer $\boldsymbol{O_l}$.

The consequence is that under the given conditions, for observers $\boldsymbol{O_e}$ the particles would no longer meet in the middle of the acceleration detector, just as it has to apply also to him.

This discrepancy could not even be completely eliminated by a transformation leading to a contraction or dilatation of the spatial coordinate, and thus to a change in the detector length.

The incongruence can only be eliminated by postulating that, from $\boldsymbol{O_e}$ observer's point of view, the particles are not at the same time in the entrance of the detector of the accelerator.

This fact indicates a dependence of simultaneity on the choice of the reference system.

From this it can be concluded that the new transformation must not only affect the spatial, but also the temporal coordinate.

Therefore, this implicitly confirms the relativistic postulate of the existence of a local time dependent on the velocity of the frame of reference.

This second consequence is even more serious than the first, which leads to the rejection of the Galilean transformation for the spatial coordinate, because it concerns the untenability of Newton's conception of an absolute time.

It should be noted that at this stage of the study there is still a lack of the necessary physical knowledge to derive the Lorentz transformations, which provide the correct relations for the temporal and spatial coordinates as a function of the velocity.

From the relativistic velocity-addition formula, the principle of the constancy of the speed of light for any relative speeds between light source and receiver must first be proven. Only then the physicist does have the necessary prerequisites for the derivation of the transformations.

One would get the following results:

After the Lorentz transformations observer $\boldsymbol{O_e}$ perceives a time dilatation[1].

For the length of the detector observer $\boldsymbol{O_e}$ instead establishes a contraction represented by the relationship:

$$\boldsymbol{l_e = l\sqrt{1 - \frac{v^2}{c^2}}}$$

The same length contraction in the direction of motion should also be perceived by the observer $\boldsymbol{O_l}$ for the colliding particles.

For example, protons should no longer be spherical, but like an ellipsoid.

For velocities, which are given here as fractions of the speed of light, the proton[2] would have to appear as shown in Fig. 12, according to the Theory of Relativity.

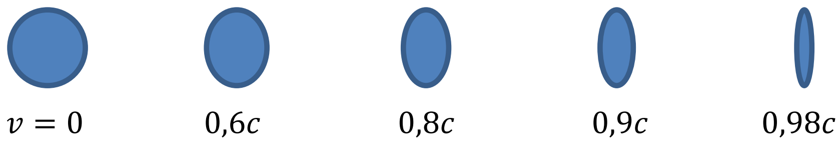

Fig. 12

With the Large Hadron Collider, the most powerful particle accelerator ever realized, protons can be accelerated up to an energy of 14 TeV.

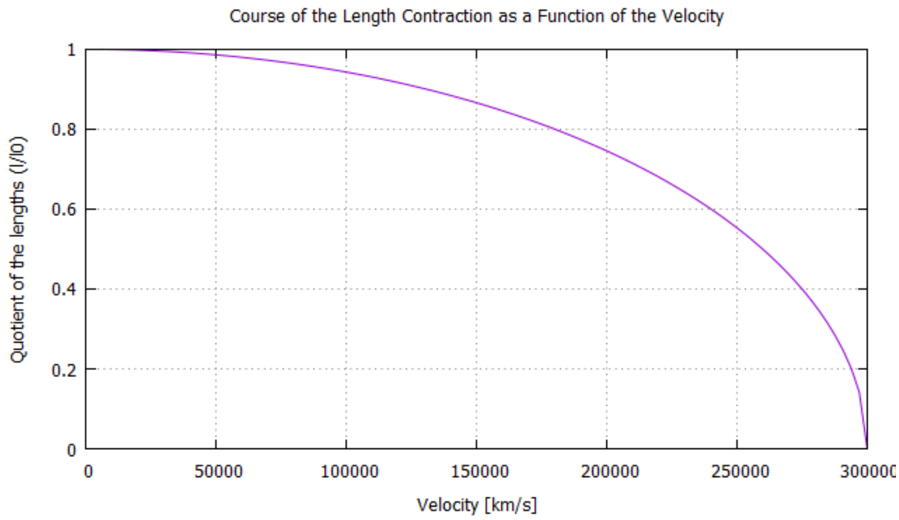

Fig. 13

This energy corresponds to a velocity $\boldsymbol{v_p}$ equal to 99.9999991% of the speed of light.

At this velocity $\boldsymbol{v_p}$ the proton would have virtually no dimension in the direction of movement.

[1] A simple derivation of time dilation can be found in the "thought experiment of the light clock" by Gilbert Newton Lewis and Richard C. Tolman.
[2] Here, the proton is considered as a classical particle, without considering possible quantum mechanical aspects

We will see in chapter 11 how relativistic length contraction can be derived from a thought experiment using the law of conservation of energy.

> In this section it was shown that, assuming the invariance of simultaneity, the measurements of time intervals taken by observers in relative motion to one another do not agree. Although at this stage of the study we do not yet have the physical means to quantitatively confirm the results of the special theory of relativity, it has meanwhile been shown that time depends on the choice of the reference system.