13 Constancy of the Speed of Light

The speed of light occurs as a constant c in many physical formulas. One of these is the relation of the momentum $\boldsymbol{p} = \boldsymbol{E}/\boldsymbol{c}$ of the light radiation. It turns out that with a stationary light source the speed of light is constant for every frequency from radio waves to gamma rays. Constancy for all frequencies does not violate the laws of classical physics.

This chapter is not about this concept of constancy, but about the fact that the speed of light remains constant even with a moving light source and between all inertial systems, as the experiment by Michelson and Morley shows.

Since this constancy applies to every relative speed between light transmitter and receiver, it is no longer in accordance with the Galilei transformation and thus conflicts with a fundamental principle of classical mechanics. That is also the reason why the concept "constancy of the speed of light in the relativistic sense" eludes the intuition of the human mind all too easily. But what is so special about this concept to cause a revolution in physics with the known consequences: time dilation, length contraction and the famous paradoxes? In this chapter we want to try to provide clarity using a concrete example:

Imagine a spaceship that moves relative to an observer $\boldsymbol{O_a}$ (index "a" stands for "outside the spaceship") with a constant, uniform speed $\boldsymbol{v}$. In the middle of the spaceship, a device simultaneously shoots two bullets $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$ at the same speed $\boldsymbol{v_k}$ ($\boldsymbol{v_k} > \boldsymbol{v}$; $\boldsymbol{v_k} \ll \boldsymbol{c}$). Bullet $\boldsymbol{k_1}$ is shot in the direction of movement of the spaceship, the other bullet $\boldsymbol{k_2}$ in the opposite direction. From the point of view of an observer $\boldsymbol{O_i}$ (index "i" stands for "inside the spaceship"), who is resting in the spaceship, the bullets reach the anterior and rear ends of the spaceship at the same time. But what does the situation look like from the point of view of the observer $\boldsymbol{O_a}$ in relative movement to the spaceship? During the flight of $\boldsymbol{k_1}$ the spaceship moved forward. Thus, the front end has moved away from the starting position of $\boldsymbol{k_1}$. Which bullet then, from the point of view of observer $\boldsymbol{O_a}$, reaches the target first?

The correct answer is that for observer $\boldsymbol{O_a}$ too both bullets reach their targets at the same time, because $\boldsymbol{O_a}$ measures not only a longer path for bullet $\boldsymbol{k_1}$, but also a higher speed than for bullet $\boldsymbol{k_2}$. Because of the relative speed of the space shuttle, observer $\boldsymbol{O_a}$ measures the speeds $\boldsymbol{v_k} + \boldsymbol{v}$ for bullet $\boldsymbol{k_1}$ and $\boldsymbol{v_k} - \boldsymbol{v}$ for bullet $\boldsymbol{k_2}$, in accordance with the addition theorem of the speeds from the Galilean transformation. Velocities and lengths then compensate each other and thus the running times of the bullets are the same. So, at low speeds there is simultaneity of the events for all observers.

But let us see what would happen if the device in the middle of the spaceship, instead of the bullets, would emit two light quanta (or photons) in opposite directions. We assume a much higher velocity $\boldsymbol{v}$ for the space shuttle than in the previous example. Analogously for example with the bullets, observer $\boldsymbol{O_a}$ would have to measure the velocities $\boldsymbol{c} + \boldsymbol{v}$, and $\boldsymbol{c} - \boldsymbol{v}$ for the photons. But this is not the case: As it is confirmed by experiments, observer $\boldsymbol{O_a}$, like $\boldsymbol{O_i}$, measures exactly the same value $\boldsymbol{c} \cong 300000\ Km/s$ for the velocities of the photons in both directions. This makes the failure of the Galilean transformation obvious and the necessity of a new transformation for space and time recognizable: a transformation is required which takes into account the invariance of the speed of light.

Now it should be clear what is meant by the term constancy of the speed of light in a relativistic sense: unlike the speed of mass bodies, the speed of light is the same from the point of view of all observers, regardless of their relative movement to the light source and to each other. In our case this leads to the following consequence: for observer $\boldsymbol{O_i}$, both photons reach the walls of the space shuttle at the same time. For observer $\boldsymbol{O_a}$, the photon emitted in the direction of travel reaches the destination later because, at the same speed $\boldsymbol{c}$, it must cover a longer distance than the other photon. So, there is no longer simultaneity of events for both observers. In cases like this one speaks of a relativity of simultaneity. In doing so, the physicist realizes that the concept of time flowing in the same way for all observers, as Newton had imagined, no longer makes sense.

At the end of the nineteenth century, physicists were still intimately familiar with the concept of absolute time and space.

When Michelson and Morley announced the results of their experiments in 1887, the scientists around the world were therefore confronted with a great surprise: The experimental observations conflicted with the principles of mechanics, because the experiment on Michelson's interferometer proved that the speed of light in vacuum is always constant, regardless of the resting or moving state of the light source.

From this knowledge the need arose to lend the natural phenomenon of the constancy of the speed of light the attribute of a fundamental physical postulate.

On the other hand, physicists felt that this postulate was not in accordance with Newton's laws. As a consequence of this conviction, they refrained from attempting to explain the constancy of the speed of light on the basis of Newton's mechanics within the framework of the laws of classical physics.

Rather, scientists came to believe that it was necessary to develop a new physical theory.

The birth of the Theory of Relativity is therefore closely linked to the assumed incompatibility of Newtonian mechanics with the natural phenomenon of the constancy of the speed of light.

However, we are now able to prove this natural phenomenon theoretically by means of the derivation of the velocity addition carried out in the tenth chapter.

For a better overview, here is a short summary of the entire path leading to the derivation of the velocity addition formula:

- The relation of the velocity addition (10.6) was derived from the application of conservation laws to the collision of two particles.
- For the energy balance, the total energies of the particles (i.e., the sum of their kinetic and internal energy) were used.

- The formula for the total energy of a particle (6.5) was derived in the sixth chapter by using the relation (5.4), which expresses the dependence of mass on velocity.
- In the fifth chapter, however, it has been shown that the relation of the dependence of mass on velocity (5.4) is a direct consequence of the second law of dynamics and the *Mass–Energy Equivalence Principle*.
- The energy-mass equivalence principle was finally derived in chapters 3 and 4 using only classical physics.

The conclusion of this argumentation is that in this work the proof of the relativistic addition formula of velocities was provided without the use of the postulate of the constancy of the speed of light.

At this point, the velocity addition formula (relation 10.6) enables us to prove the principle of the constancy of the speed of light in a purely theoretical way.

For this purpose, we assume a light source that moves relative to an observer. He can use equation (10.6) ...

$$\boldsymbol{v_{12}} = \frac{\boldsymbol{v_1} + \boldsymbol{v_2}}{\boldsymbol{1} + \frac{\boldsymbol{v_1 v_2}}{\boldsymbol{c^2}}} \qquad (10.6)$$

... to calculate the relative velocity $\boldsymbol{v_l}$ of the emitted light:

If $\boldsymbol{v_1}$ is replaced by the speed $\boldsymbol{c}$ of the light from a stationary light source and $\boldsymbol{v_2}$ by the velocity $\boldsymbol{v_q}$ of the light source, then:

$$\boldsymbol{v_l} = \frac{\boldsymbol{c} + \boldsymbol{v_q}}{\boldsymbol{1} + \frac{\boldsymbol{c v_q}}{\boldsymbol{c^2}}} \qquad \Rightarrow$$

$$\boldsymbol{v_l} = \frac{\boldsymbol{c} + \boldsymbol{v_q}}{\frac{\boldsymbol{c} + \boldsymbol{v_q}}{\boldsymbol{c}}} \qquad (13.1)$$

It can now be shown easily that equation (13.1) always assumes the solution $\boldsymbol{v_l} = \boldsymbol{c}$ for any velocity $\boldsymbol{v_q}$ of the light source.

This proves that the speed of light in each inertial system, regardless of its state of motion, is always the same.

Looking more closely at the overall approach that led to this evidence, it can be stated that:

The constancy of the speed of light can be proven purely theoretically, i.e., also without the use of experiments, but for the confirmation of these.

> By using the addition formula of velocities, the principle of the constancy of the speed of light can be theoretically proven. From this point of view, the theorem of the constancy of the speed of light is not a postulate, but a principle provable by the laws of physics.